

## **Bildung in a New Context in Danish University Teaching with some Remarkable Results**

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### **Abstract**

In this paper we will look at the pedagogical and didactic concept of Bildung and how Bildung has been used as an important ingredient of European university teaching during the last 200 years. We will also shortly look at the modern university teaching where Bildung in some important respects has been abandoned and even abolished. This, however, has had many bad consequences such as higher rates of failure. In particular, these bad consequences have been seen very clearly at The Department of Economics of The University of Copenhagen. A reorganization of the teaching of mathematics at this department began during the autumn term of 2007 where some elements of Bildung in a new modern context were included in the lectures. This had a remarkable impact. The students became more interested in the teaching of mathematics and they became more engaged and active. Their study activity improved and the rate of failure began dropping. From 2010 the classes of mathematics were also dramatically reorganized such that Bildung and study techniques were integrated elements of the class teaching. How this was practically done will be the main focus of this paper and it will be shown that Bildung is central and important if we want to develop university teaching to such a level that the enlightenment of the academic world can continue. Furthermore it will be shown that this new way of teaching mathematics at The Department of Economics has had a substantial impact, dropping the rate of failure of the summer exams of mathematics in June 2011 to its lowest level hitherto and also improving the marks of mathematics generally.

### **The Concept Of Bildung**

Bildung is a German word. There is no English word for Bildung, but often Bildung is translated as “self-cultivation”, “self-formation”, or “self-perfection” (Bruford 1975, Bohlin 2008). The German tradition of Bildung goes back to the 17<sup>th</sup> century when several theological philosophers focused on the ideal or obligation, that a devout Christian should seek to cultivate his or her talents and intellectual power according to the image of God (Latin: “Imago Dei”), which was innate in one’s own soul (Korsgaard 2004, Stagstad 2003). In the 18<sup>th</sup> century the concept of Bildung became closely related to education but in a very specific way. Knowledge and education should form one’s soul and develop one’s intellect in accordance with God’s creation of man in his own image. As a consequence of this ideal Bildung, was, and is still, considered as a pedagogical and social norm for personal competency such that Bildung refers to both the process and the aim: “To become and to be self-cultivated” (In German: gebildet) (Borgnakke 2008). This has also been expressed in the

following way by the Swedish author Ellen Key (1849–1926): “Bildung is what is left in your soul when you have forgotten everything you have learnt“. (Borgnakke 2008).

In relation to insight, knowledge, and development of personal competency Bildung became closely connected to the teaching and educational system but Bildung is also connected to upbringing and socialization in general (Borgnakke 2008). Furthermore, in the second half of the 18<sup>th</sup> century Bildung became a term with some political connotations and associated with a personal liberation of one’s mind from irrelevant traditions and superstition.

All kinds of education form our minds to some extent. The works by the Prussian philologist, lawyer, educational reformer, and politician Wilhelm von Humboldt (1767–1835) brought Bildung into correspondence with a new ideal of education: Education (e.g. university teaching) was imagined as as a lifelong and progressing process where personal and social skills grow and expand continuously. (Bruford 1975, Bohlin 2008).

In 1791–1792 von Humboldt wrote the following that might be considered as a definition of Bildung: “The true end of Man, or that which is prescribed by the eternal and immutable dictates of reason, and not suggested by vague and transient desires, is the highest and most harmonious development of his powers to a complete and consistent whole”. (Humboldt [16] 1791–1792, Chapter 2).

Swedish researchers Helen Avery and Monne Wilborg, point out that: “Bildung stands for an educational ideal that can be traced back to the Greek tradition, and which is informed by aspirations of humanism, neo-humanism, and enlightenment”. These ideals have profoundly influenced modern western educational culture, and informed the development of higher education. (Avery and Wilborg 2011).

Although Bildung and enlightenment are two different concepts it is natural to consider enlightenment in connection with higher education as mentioned above (Avery and Wilborg 2011). One of the most renowned definitions of enlightenment was given by the famous German philosopher Immanuel Kant (1724–1804) in 1784 (Kant [20] 1784, Slagstad 2003, Olesen [41] 2010). In his well known article “Beantwortung der Frage: Was ist Aufklärung?” (What is Enlightenment?) he wrote:

Aufklärung ist der Ausgang des Menschen aus seiner selbstverschuldeten Unmündigkeit. Unmündigkeit ist das Unvermögen, sich seines Verstandes ohne Leitung eines andern zu bedienen. Selbstverschuldet ist diese Unmündigkeit, wenn die Ursache derselben nicht am Mangel des Verstandes, sondern der Entschliessung und des Muthes liegt, sich seiner ohne Leistung eines andern zu bedienen. Sapere aude! Habe Muth, dich deines eigenen Verstandes zu bedienen! ist also der Wahlspruch der Aufklärung.

In English translation this is:

Enlightenment is man’s release from his self-incurred tutelage. Tutelage is the incapability to use one’s own understanding without the guidance of another. Such tutelage is self-imposed if its cause is not lack of intelligence, but rather

a lack of determination and courage to use one's intelligence without being guided by another. Sapere aude! Be courageous and use your own intelligence! This is the motto of enlightenment.

A person being enlightened dares to know (Latin: Sapere aude) and this personal courage is the prerequisite for the self-formation of one's mind that is also so important in modern learning (Olesen [41] 2010).

Kant was brought up in a pietistic home and he was influenced by the German mathematician Martin Knutzen (1713–1751), the English philosopher John Locke (1632–1704), and the Franco-Swiss philosopher Jean-Jacques Rousseau (1712–1778). In 1690 John Locke published his great work “An Essay Concerning Human Understanding” (Hartnack [12] 1965), and he pointed out that when a child is born its mind is totally empty, just as a piece of white paper is blank. According to John Locke, our knowledge is determined only by experience and is derived from our sense perception.

This point of view was also central to Rousseau when he wrote his pedagogical masterpiece “*Émile ou de l'éducation*” in 1772 (Korsgaard [46] 2003, Korsgaard [28] 2004). But to Rousseau a child had to be brought up and to be taught or rather guided on its own individual conditions to become an educated person and a free citizen in a given political community.

To Kant, being influenced by pietism, both Locke's and Rousseau's thoughts were fundamental to his own definition of an enlightened person (Kant [20] 1784), and to reach the educational aims of the individual he had the opinion that upbringing and education of children and young people must be carried out using firm discipline. Duty and industry, the two well known pietistic virtues, were clearly central in all education according to Kant (Korsgaard [28] 2004).

In his work “*Über Pädagogik*” Kant pointed out that nature of man should not be considered as some given resistance against education that had to be defeated but rather a basis and a condition for personal and educational development (Kant [25] 1803). Furthermore, education and teaching must be considered as a specific art or science where the students are highly respected and guided firmly by competent teachers. This was a specific sort of humanism that gained great importance when *Bildung* later was implemented into high school and university teaching in several European countries during the first half of the 19<sup>th</sup> century (Koch [26] 2003, Koch [27] 2004).

Both to Kant and many other philosophers of his time enlightenment and *Bildung* should not only be considered in relation to the individual and the intellect, but also as a cultural and social component in the political community with great importance to mankind. This point of view is the fundament of what we might call “the educational theoretical trinity”, often being illustrated as the sides of an equilateral triangle: (1) *Bildung* is first of all an aim for all individuals and it is closely related to concepts such as duty, industry, freedom, emancipation, autonomy, individualism, responsibility, reason, and knowledge. (2) The relationship between the individual and everything that is outside the individual, i.e. one must have profound knowledge about nature and about the eternal laws that govern nature and the

whole universe, tolerance and respect for other people, humanism, and objectivity. All this is closely related to the aptitude of individual judgment, our way of thinking, and our self-formation. 3) The relationship between the individual and society. How does one behave to be a responsible citizen in a given political unity? How does one communicate with other individuals? These questions concern political and personal morality. To Rousseau the answer to these questions was the so-called “social treaty”, and to Kant the answer was the new concept of citizenship. Today these three aspects of Bildung are often summarized using the three words: Knowledge, thought and communication, and “the educational trinity” (in German: Bildungsdreieck) is still central in modern didactic and pedagogical philosophy (Bohlin 2008, Hartnack [13] 1966, Korsgaard [46] 2003, Korsgaard [28] 2004, Olesen [41] 2010).

Among all the philosophical works Kant wrote, three especially have had a central impact on our understanding of Bildung. These three are his famous “critiques”: “Kritik der reinen Vernunft”, Eng. “Critique of Pure Reason”, (Kant [21] 1787), “Kritik der praktischen Vernunft”, Eng. “Critique of Practical Reason”, (Kant [22] 1788), and “Kritik der Urteilskraft”, Eng. “Critique of Judgment”, (Kant [23] 1790). In “Kritik der reinen Vernunft” he outlines his theory of knowledge, in “Kritik der praktischen Vernunft” he deals with general morality and the human will, and in “Kritik der Urteilskraft” Kant establishes a theory of human judgment.

These three issues became very central in Danish Bildung tradition during the first part of the 19<sup>th</sup> century, when the Danish physicist Hans Christian Oersted, who had discovered electromagnetism in 1820, talked and wrote a lot about “the true, the good, and the beautiful”. (Christensen [7] 2009). Also in Norway the same Bildung tradition as the Danish was formed in the 19<sup>th</sup> century. (Slagstad 2003).

The Prussian politician and lawyer Wilhelm von Humboldt (1767–1835) developed his own interesting theory of Bildung, and he had an essential pedagogical point of view which became fundamental for development of the modern university: In 1810 he established the famous Berlin University where the principles of teaching were naturally based upon his and Kant’s theories of Bildung. Also teaching and research now became closely related in what we call “The Humboldtian University” (Humboldt [17] 1792, Kant [24] 1798).

This new concept of university teaching and research based upon Bildung was spread from Berlin to many European countries and also to Denmark, and the University of Copenhagen was transformed from an Aristotelian-Scholastic to a Humboldtian University with great success (Helmholtz 1877, Huxley 1876, Jaspers 1923, 1946, Korsgaard [46] 2003, Korsgaard [28] 2004, Nordenbo 1980, Olesen [41] 2010).

## **Historical Development**

The new philosophical ideas of the Enlightenment Era caused an interesting development in the united Danish-Norwegian monarchy. In order to strengthen and modernize the teaching in the high schools and at the universities in Copenhagen and Kiel a royal commission was appointed in 1790 under the guidance of Duke Frederik Christian of Augustenborg (1765–

1814) (Korsgaard [28] 2004, Olesen [41] 2010). The work of the commission resulted in the establishment of a new pedagogical faculty at the University of Copenhagen in 1800 and the philanthropist and philosopher Levin Christian Sander (1756–1819) was appointed as professor at this faculty. The main purpose of the pedagogic faculty was to educate high school teachers who were acquainted with the new enlightenment ideas of teaching and *Bildung*, and in 1809 a reform of the high schools and the universities was accomplished such that elements of *Bildung* were introduced in the teaching (Nordenbo 1980, Korsgaard [28] 2004, Olesen [41] 2010, Steffensen 1979). But the economical and national disaster after the Napoleonic Wars, where the united Danish-Norwegian monarchy was split up in 1814, stopped this reform process for a couple of decades (Olesen [41] 2010).

In the 1820s the German psychologist Johann Friedrich Herbart (1776–1841) developed a new theory of *Bildung* and education in which didactics and teaching methods were included. He invented the concept of “*Bildsamkeit*”, i.e. the ability for a human being to be educated and brought up (Nordenbo 1980, Koch [27] 2004, Olesen [41] 2010).

At the same time a new, very critical debate about education and teaching at Danish high schools and at the University of Copenhagen broke out. Reforms of the educational system were absolutely necessary if the Danish society was to be able to meet all the new challenges that came from the fast social and scientific development taking place in most European countries. Especially it seemed essential to include the natural sciences in high school teaching and give up Latin as the main subject. Latin was a “dead” language and instead the students had to learn about different other topics that belonged to the real world, such as physics and chemistry. This point of view was put forward by many prominent researchers and teachers but especially Christian Lütken (1791–1856), who was a lecturer at the Academy of Soroe. Lütken emphasized in 1830 that high school teaching was old fashioned and that reforms had to be carried out quickly (Krarup 1955, Nordenbo 1980). He was a close adherent to the new ideas of *Bildung*, and he stressed that the students at the high schools would obtain insight and *Bildung* if they were taught natural sciences as well as Greek and Latin. Shortly afterwards two highly and internationally respected professors from the University of Copenhagen, the physicist Hans Christian Oersted (1777–1851) and the philologist Johan Nicolai Madvig (1804–1886), began working for a new Danish educational system based upon the three aspects of *Bildung* and the neo-humanistic philosophy. (Krarup 1955, Christensen [6] 2009, Olesen [41] 2010).

In 1828 Johan Nicolai Madvig was appointed as professor of classical philology at the University of Copenhagen and soon thereafter he became one of the leading philologists of Europe. His linguistic research was as remarkable as Oersted’s was on physics and he was internationally acknowledged. Madvig was strongly influenced by the new humanistic philosophical school and he supported and joined Kant’s and Humboldt’s educational ideas of *Bildung* and teaching. Just as H. C. Oersted did, Madvig wanted to modernize the Danish educational system, especially the teaching and the syllabuses of the high schools and he thought the university needed to be changed considerably (Boserup 1992, Krarup 1955, Petersen 1993, Christensen [6] and [7] 2009, Olesen [41] 2010).

In Madvig's opinion teaching at high schools should no longer be based upon Latin. Although he was a famous philologist he thought that Latin, being a "dead" language, should not dominate high school classes. European culture, philosophy, mathematics, and natural sciences had their origin in ancient Greece and hence it was important to teach Greek culture and philosophy from the classical epoch. The main purpose was to spread knowledge about the cultural development that was the basis of European civilization and the new enlightenment. Furthermore, Madvig also wanted most of the basic first year university courses to be transferred to the high schools and he wanted a unified high school curriculum with pedagogical *Bildung* at its center. In this way the high school education would be improved, and this would cause an essential impact on the university studies because the first year students would already have acquired some useful and basic academic abilities before they were matriculated at the university. As a consequence of this fact the general academic level would increase considerably and in all respects be influenced of *Bildung*. (Krarup 1955, Korsgaard [46] 2003, Korsgaard [28] 2004, Christensen [7] 2009, Olesen [41] 2010). This point of view was fully supported by H. C. Oersted (Christensen [7] 2009).

In the academic world Madvig obtained a very central position. He was elected rector of the University of Copenhagen six times and he was also appointed to a very influential political position after the democratic breakthrough in the spring of 1848. He was elected to the new Danish parliament and he was appointed minister of education in November 1848. Therefore, Madvig had in place extremely good academic and political prerequisites to reform the Danish high schools and the university, and he set to the task (Krarup 1955).

In 1848 a new study of economics was established at the university, such that teaching the social sciences was strengthened. Before 1848 economics has just been a minor discipline of other studies such as law or humanities. In 1850 mathematics and the natural sciences were organized in a new independent faculty (Phil 1983) and a first year course called "Philosophicum", common for all university students, was established (Witt-Hansen 1970). "Philosophicum" was a course on the philosophical disciplines: Logic, psychology, and history of philosophy. Other philosophical topics were now transferred to the high schools and furthermore the high school teaching was modernized, such that the natural sciences and Greek culture became a considerable part of the high school syllabus (Krarup 1955). All this was carried out such that the teaching was in full accordance with the ideas of *Bildung*, and Madvig's educational reforms were strongly approved by H. C. Oersted. The didactic revolution led by Kant, Humboldt and several other enlightenment philosophers had now been accomplished in Denmark. Interestingly enough, similar educational reforms were accomplished in Norway.

The new high school and university teaching based upon *Bildung* was a very big success. Danish scientific research flourished. Names such as Harald Westergaard, Julius Petersen, Niels R. Finsen, Niels Bohr, Harald Bohr, Aage Bohr, August Krogh, Johan Ludvig William Valdemar Jensen, Børge Jessen, K. A. Jensen, and Bent Fuglede are all world famous Danish scientists, and they were taught according to Oersted's and Madvig's principles of *Bildung* during their university studies in Copenhagen.

But things changed considerably after 1990. Many more young people graduated from the high schools and many more students were matriculated at the universities. Academic education was no longer a privilege for an elite but rather an opportunity for a big and rapidly increasing fraction of all young people. The former Humboldtian universities were transformed into mass universities and scientific research was spread from a university monopoly to many other private and public institutions. Also the elitist content in the syllabuses of higher education was swept away. The so-called Bologna process was implemented in many European countries, including Denmark, and all Danish university studies were cut shorter such that approximately 20 per cent of the syllabuses disappeared. This implied that the candidates' academic qualifications, personal aptitude, and knowledge decreased considerably. The natural sciences and mathematics were hit profoundly. (Damberg et al 2006, Olesen [35] 2008, Olesen [36] 2009, Olesen [37] 2009, Olesen [39] 2009, Olesen [41] 2010).

Strangely, this was not considered being a big political problem by many politicians and there were educational authorities that considered this development as great progress. The ideals of *Bildung* and the traditional academic virtues were old fashioned, they said. Instead the new political aim was to produce as many candidates as possible, no matter what qualifications they had earned. Educational institutions were run using management and business methods as if they were manufacturing companies or heavy industries. Especially after 2000 this management process was increasingly implemented in university and high school administration (Slagstad et al. 2003: Chapter 17 "Teknokulturell Danning" by Lars Løvlie, Olesen [41] 2010).

The gap between the high school syllabuses and the introductory level at the universities grew bigger and bigger, and in 2005 new high school legislation was passed and make this gap even bigger. Many Danish educational authorities were still talking a lot about *Bildung*, which they didn't know anything about, and no *Bildung* actually remained at the high school or university level (Olesen [37] 2009, Olesen [41] 2010). In fact: *Bildung* was abolished in all higher education.

Since 1970 an introductory and compulsory course of mathematics was included in the first year syllabus at The Department of Economics in Copenhagen. Until 2005 the rate of failure for the first year exam of mathematics had been quite stable in the interval from 15–20 per cent, but in 2006 the rate began increasing and in 2007 it reached 34 percent. (Olesen [33] 2007, Olesen [34] 2008, Olesen [35] 2008, Olesen [36] 2008, Olesen [38] 2009, Olesen [39] 2009, and Olesen [41] 2010). This was unacceptable.

### **More Mathematics**

In autumn 2007 it was decided that the teaching of mathematics at The Department of Economics had to be reorganized and changed such that the students at the beginning of their first term were taught all the elementary mathematics they no longer learned at high school, and this required that the lectures were prolonged by 50 per cent—from 2 to 3 hours a week. Although this was absolutely necessary it was not, in fact, sufficient. Something more had to

be done such that the students became more motivated, active and studious. Here it was obvious to the study leader of The Department of Economics and to the lecturer of mathematics that some elements of classical Kant-Humboldtian Bildung had to be re-introduced in the teaching of mathematics.

The main purpose for doing so from the very beginning of the first term was to make the students personally engaged, more studious, and highly motivated. They were starting a study of economics and they had to understand why mathematics plays such an important role in modern economic theory. Therefore right from the beginning of the term students are introduced to aspects of science theory and the historical developments of the past 300 years where mathematical methods have become increasingly crucial to the understanding of economic theory.

At the beginning of the reform process, in the spring of 2008 this was done a little heuristically but was fully implemented in the lectures from summer 2009. Starting with the axiomatic method of mathematics, logic, abstract set theory and the hypothetical-deductive method of the natural and empirical sciences such that the philosophical basis had been presented we continued talking about the so-called “South Sea Bubble” in 1720 where the British stock market broke down. Sir Isaac Newton and the composer Georg Friedrich Handel lost a lot of money, and when Newton was asked: “What can be done”, he just answered: “I can calculate the movement of the stars but not the madness of men”. Was it really impossible to apply mathematics to economic problems? The students had to consider this question for a week, and of course they were told the answer in the following lectures.

However, it wasn't impossible to apply mathematical methods and theory to solve economic problems, but we had to verify this assertion. And we went on to describe historical developments: The Swiss mathematician Daniel Bernoulli (1700–1782) introduced in 1738 the logarithmic utility function with great success, and now the students might understand that economics can be treated and described scientifically in a similar way as physics. Also in 1738 Daniel Bernoulli solved the famous Saint Petersburg paradox which is a difficult and complicated stochastic economic problem. Then we continue talking about the physiocratic theories, especially the French school (Quesnay and Turgot), where a nation's economy is considered as a system that can be described using exact terms. One of Turgot's close friends was the French mathematician and philosopher Marie-Jean Antoine-Nicolas Caritat Marquis de Condorcet (1743–1794). He began research in the application of mathematical methods to economics and other social sciences. This is also told the students to describe how scientific economic theory developed from the end of the 18<sup>th</sup> century. Condorcet's interesting theory of voting and his personal claim for woman suffrage in 1790 was an important part of the mathematical Bildung, and telling all this to the first year students makes them engaged, studious, and enthusiastic. (Olesen [32] 2007, Olesen [41] 2010).

In the next lectures elementary mathematics is described but we also continue describing the close historical connection between economics and mathematics. An interesting anecdote concerns the famous riddle about crossing the seven bridges in the East-Prussian city of Königsberg. This classical mathematical problem was solved by the Swiss mathematician Leonhard Euler (1707–1783) in 1735, and furthermore he created a new

general theory for crossing any number of bridges. At first glance this problem has nothing to do with economic theory, but then the students are told that Euler's theory was the beginning of graph theory which plays a central role in modern network analysis (Hopkins 2007, Olesen [32] 2007, Sandifer 2007, Wolff [49] 1963). At this stage the students also need to know that the Danish mathematician Julius Petersen (1839–1910), who was interested in scientific economics and who even developed some mathematical models on distribution of goods in 1873, made some important contributions to graph theory. (Davidsen 2001, Lützen 1992, Petersen 1891).

Although it has nothing to do with neither mathematics nor economics it is told in the next math lectures that Euler worked most of this life outside Switzerland; at the imperial academy in Saint Petersburg during the years 1727–1741 and 1766–1783 and at the royal scientific academy in Berlin from 1741–1766. This is done to make the students understand the importance of international relations in scientific and academic work, and that this is not a new phenomenon. Things like that are also very important in the way *Bildung* is introduced in the modern university teaching.

It is also very important to tell the students about Lagrange's theory from 1788 on determination of extreme points under some given constraints. This was a new mathematical theory developed by the French mathematician Joseph-Louis Lagrange (1736–1813) and at the end of the 18<sup>th</sup> century applied with great success to complicated celestial mechanical problems, but in 1876 the Danish mathematician and economist Harald Westergaard (1853–1936) applied the Lagrange theory to a problem concerning marginal utility. Here the students are told an example concerning how pure mathematical research many years later may become central in solving practical economic problems (Davidsen 2001).

The historical review of this first year course of mathematics is ended by telling the students about the mathematical finance theory that was introduced in 1900 by the French economist Louis Bachelier (1870–1946) when he published his thesis “*Théorie de la Spéculation*” (Olesen [32] 2007, Growers 2008). Then we talked about the important scientific principle of generalization and building up new theories as a common large umbrella including several different and less general theories. After this we began studying pure mathematics such as calculus in general and linear algebra. And here it is very important not to use any electronic calculators or any computer related equipment. The students have to learn the proofs of all the lectured theorems and propositions and they have to be precise in their calculations, argumentations, and formulations such that they can handle mathematical problems correctly. In this way they gain mathematical aptitude and they strengthen their ability to analyze complex problems. (This kind of *Bildung* we specifically might call mathematical *Bildung*).

We hereby see that two of the three items in the classical “trinity of *Bildung*” have been considered in this kind of teaching and the students really liked it. They were clearly more engaged, more studious and enthusiastic than ever before, and they began discussing what their study was all about (Olesen [41] 2010). That was extremely important because in this way they began their own personal self-formation at an academic level. It is interesting to

notice, that recently a similar math teaching program has been launched in Brazil with great success (Gouvêa 2011).

But we had to go much further. The last item in the classical theory of Bildung had also to be taken into account to complete the new teaching efforts. We had to do something that appealed to the students' social competences, and here it was very important to look closely at the classes where concrete mathematical problems are solved. Until September 2010 the students attending these classes had been guided carefully with respect to solving mathematical problems by a teaching assistant, but now we tried to engage the students more by focusing on group work and by integrating some important aspects of study techniques in the classes. They had to be responsible for their own learning such that they later (both in their further studies and after their graduation) would be able to learn even advanced theories without any instruction or guidance. We had to encourage them to study more consciously in groups and to give them the ability to make further and more advanced studies on their own.

All students should attend study groups consisting of 3–4 people and in the groups they had to try solving all the problems listed in the study plan before taking the classes. If they hadn't done their homework they were told very clearly that it was not interesting for them or their fellow students if they attended the classes. In fact, they had to be personally responsible and reliable.

During their classes the students were asked to present their solutions on the blackboard or to present a theorem from the textbooks. Furthermore the students were forced to discuss their personal study program every week during the term both with each other and with the teaching assistant. They also had to discuss and consider feedback after their presentations on the blackboard. This was a very big challenge both for the students and for the teaching assistants. But the students had to work hard, concentrated, and conscientiously to meet the requirements and in this way they achieved a high level of study activity.

Did all these efforts have any impact? They certainly did! The rate of failure for the first year exams in mathematics dropped from 34 per cent in June 2007 to only 7.2 per cent in June 2011. Let us look a little more closely at this very remarkable development. And here it must be clearly emphasized that the difficulty of the problems at the written exams was absolutely unchanged.

### **Evaluation Criteria and Exam Results**

All Danish university exams are assessed using an international scale consisting of the grades: 12, 10, 7, 4, 02 (which are passing grades), and 00, and – 3 (which are failure grades). The top grades are 12 and 10, the middle grades are 4 and 7, and the grade 02 is given for just passing the exam. The failure grades are 00 and – 3, where – 3 is only given if almost nothing is correct. Compared to the ECTS-scale we have that 12 = A, 10 = B, 7 = C, 4 = D, 02 = E, 00 = FX, and – 3 = F.

How to translate from percentage of scores to the grades, the following scale of equivalence is used:

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|                   |               |                |                |                |                |                |                 |
|-------------------|---------------|----------------|----------------|----------------|----------------|----------------|-----------------|
| <b>Percentage</b> | <b>0 – 10</b> | <b>11 – 49</b> | <b>50 – 59</b> | <b>60 – 67</b> | <b>68 – 83</b> | <b>84 – 91</b> | <b>92 – 100</b> |
| <b>Grade</b>      | <b>- 3</b>    | <b>00</b>      | <b>02</b>      | <b>4</b>       | <b>7</b>       | <b>10</b>      | <b>12</b>       |

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### **The Results Of The Exams In June 2008 – 2011**

#### **2008**

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|                               |            |             |             |             |             |             |             |
|-------------------------------|------------|-------------|-------------|-------------|-------------|-------------|-------------|
| <b>Grade</b>                  | <b>- 3</b> | <b>00</b>   | <b>02</b>   | <b>4</b>    | <b>7</b>    | <b>10</b>   | <b>12</b>   |
| <b>Number of students</b>     | <b>6</b>   | <b>46</b>   | <b>36</b>   | <b>29</b>   | <b>33</b>   | <b>22</b>   | <b>36</b>   |
| <b>Percentage of students</b> | <b>2.9</b> | <b>22.1</b> | <b>17.3</b> | <b>13.9</b> | <b>15.9</b> | <b>10.6</b> | <b>17.3</b> |

208 students attended the exam, 156 students passed the exam, and the rate of failure was 25.0 per cent.

The average grade of all students: 5.1 and the average of the students who passed: 6.9.

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#### **2009**

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|-------------------------------|------------|-------------|-------------|-------------|-------------|-------------|-------------|
| <b>Grade</b>                  | <b>- 3</b> | <b>00</b>   | <b>02</b>   | <b>4</b>    | <b>7</b>    | <b>10</b>   | <b>12</b>   |
| <b>Number of students</b>     | <b>1</b>   | <b>15</b>   | <b>17</b>   | <b>18</b>   | <b>38</b>   | <b>20</b>   | <b>17</b>   |
| <b>Percentage of students</b> | <b>0.8</b> | <b>11.9</b> | <b>13.5</b> | <b>14.3</b> | <b>30.2</b> | <b>15.9</b> | <b>13.5</b> |

126 students attended the exam, 110 students passed the exam, and the rate of failure was 12.7 per cent.

The average grade of all students: 6.1 and the average of the students who passed: 7.1.

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**2010**


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|                               |            |             |             |             |             |             |             |
|-------------------------------|------------|-------------|-------------|-------------|-------------|-------------|-------------|
| <b>Grade</b>                  | <b>- 3</b> | <b>00</b>   | <b>02</b>   | <b>4</b>    | <b>7</b>    | <b>10</b>   | <b>12</b>   |
| <b>Number of students</b>     | <b>1</b>   | <b>16</b>   | <b>37</b>   | <b>28</b>   | <b>25</b>   | <b>28</b>   | <b>17</b>   |
| <b>Percentage of students</b> | <b>0.7</b> | <b>10.5</b> | <b>24.3</b> | <b>18.4</b> | <b>16.4</b> | <b>18.4</b> | <b>11.2</b> |

152 students attended the exam, 135 students passed the exam, and the rate of failure was 11.2 per cent.

The average grade of all students: 5.4 and the average of the students who passed: 6.3.

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**2011**


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|                               |            |            |             |             |             |             |             |
|-------------------------------|------------|------------|-------------|-------------|-------------|-------------|-------------|
| <b>Grade</b>                  | <b>- 3</b> | <b>00</b>  | <b>02</b>   | <b>4</b>    | <b>7</b>    | <b>10</b>   | <b>12</b>   |
| <b>Number of students</b>     | <b>0</b>   | <b>15</b>  | <b>31</b>   | <b>31</b>   | <b>65</b>   | <b>29</b>   | <b>41</b>   |
| <b>Percentage of students</b> | <b>0.0</b> | <b>7.1</b> | <b>14.6</b> | <b>14.6</b> | <b>30.7</b> | <b>13.7</b> | <b>19.3</b> |

212 students attended the exam, 197 students passed the exam, and the rate of failure was 7.1 per cent.

The average grade of all students: 6.7 and the average of the students who passed: 7.2.

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Looking at these results it is quite clear that it has had a remarkable impact on the exam results since Bildung was introduced in the teaching of mathematics at The Department of Economics. And we notice that the exam results are improving each year. The rate of failure is dropping monotonously year after year and has in 2011 reached a historically low point. All this has happened as more and more of the teaching has been influenced by Bildung. Now all three aspects of the “trinity of Bildung” have been implemented, and therefore the result from 2011 is in particular interesting.

These results are very remarkable, and there is no doubt that the new way of teaching mathematics at The Department of Economics, where all three items from the classical trinity of Bildung are taken into account just as described above, has had a measurable and indisputable impact.

**Conclusion**

We have seen how the classical trinity of Bildung has been implemented into the teaching of mathematics at The Department of Economic in Copenhagen with great success. The students

achieved better exam results in mathematics and they achieved a much higher academic level in general. This is not only important for their further studies but also important for them when they have graduated. Bildung is very important for the candidates when they are employed in an academic position either at a university or in a company, a bank, or another institution employing highly educated persons.

These aptitudes are concentrated about mathematical topics of economic theory and the students' learning of mathematics but the basic tools in this way of teaching, as we have described above, are not specifically related to mathematics. These educational tools can be applied to all other academic topics, and as a consequence of this fact the study board has decided that Bildung must be an important ingredient in all lectures and classes at The Department of Economics. Bildung is in all respects a basis for life-long learning.

Now it is also very important to emphasize that if we give up Bildung in high school and university teaching, which unfortunately has already happened at many teaching institutions both in Denmark and in many other European countries, we only support the spreading of what has been called anti-enlightenment and that is a very serious problem. Then the students' knowledge will be too superficial and fragmented, and then we will not be able to competing with nations such as China, Japan and India in the long run. But teaching with Bildung included, as we have sketched here in this article, will give us more satisfied, motivated, and skilled students and furthermore candidates who are able to study complex systems and difficult topics.

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